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Phase-Space Estimate
of Satellite Coverage Time

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OUTLINE

PHASE-SPACE ESTIMATE OF SATELLITE COVERAGE TIME

by

Gregory H. Canavan

ABSTRACT

A phase-space estimate of the overlap in satellite coverage is used to evaluate impact on the time for constellation coverage. It also gives the distribution of vintages of data, the average of which is essentially the coverage time. Inclined orbits' overlaps in areas of interest makes measurements more current there.

I. INTRODUCTION

This note gives a phase-space estimate of the impact of overlaps in coverage on the time it takes a constellation of satellites to cover a specified fraction of the Earth's surface and the vintage of the resulting data.

II. CALCULATION

Each satellite in a constellation is assumed to have a transverse swath W and speed V, so that it sweeps out area at a rate W·V. In the absence of overlaps in coverage, N such satellites would search an area A in a time

$$T_0 = A/NWV. \quad (1)$$

Some satellites re-cross areas that have already been covered before the constellation covers all of A, so the actual time to cover the whole area is larger than T.

It is awkward to calculate the amount of overlap precisely, particularly for constellations with a distribution of inclinations. However, a phase-space estimate is possible and much simpler. The satellites move in deterministic motions, but the combination of their motion and that of the Earth causes the coverage to appear erratic to observers on the ground, which suggests a model in which the satellites are treated as if their motion was random.

In a time W/V, each satellite covers an area W^2. N satellites would cover area $N \cdot W^2$, for $N \cdot W^2 \ll A$. The estimate below replaces their actual motion with a statistical problem, which assumes that every time W/V the satellites' positions are changed randomly, and that each is placed on a grid of W by W squares covering A. At a given time the probability that a given square is covered is $N \cdot W^2/A$, and the probability that it is not covered is

$$p_1 = 1 - NW^2/A. \tag{2}$$

The probability that it is also not covered in the subsequent step is $(p_1)^2$; that it is not covered in the next either is $(p_1)^3$. The probability that it is not covered in M steps is

$$p_M = (p_1)^M = (1 - NW^2/A)^M. \tag{3}$$

If it is required that the probability that the area be covered at least once in M steps is specified, then the number of steps required for at least one measurement is

$$M = \ln(p_M)/\ln(1 - NW^2/A) \approx -A/NW^2 \cdot \ln(p_M), \tag{4}$$

for $NW^2/A \ll 1$, which takes a time

$$T = M \cdot W/V \approx (A/NWV)\ln(1/p_M). \tag{5}$$

The net effect of overlaps is to change the zero-overlap coverage time T_0 of Eq. (1) by a factor of $\ln(1/p_M)$. For a 0.9 probability of coverage, i.e., $p_M = 0.1$, $\ln(1/p_M) \approx 2.3$; for a 0.99 probability of coverage, $p_M = 0.01$, $\ln(1/p_M) \approx 4.6$. Thus, for high probabilities of coverage the correction is a factor of

2-4. For a probability like 0.7, which still gives timely measurements, the correction is negligible.

III. VINTAGE OF INFORMATION

At any time, for some cells the measurements will be recent, for other cells the measurements will be older. The analysis above also gives a rough indication of the vintage of the information.

A. Vintage

For a probability of coverage of 0.7, about a third of the cells would have been missed; the other two-thirds would have been measured during T. The average age of their measurements would be about T/2.

For a probability of coverage of 0.9, the satellites would have covered A about 2.3 times. About 10% of the cells would have been missed; about half of the cells would have been measured once in T; and about half would have been visited twice. Thus, about 10% of the cells would have a measurement as old as T. Half would have measurements of age about 3T/4. The other half would have ages of about T/4.

For higher probabilities of coverage, the satellites would have covered A more times in T and missed fewer cells. Thus, the vintages of the measurements would be much more recent for most of the cells. At any time the data base would have measurements of different vintages for different cells, depending on the specific constellation. The most recent information could be quite useful, as long as the data base kept track of when the measurements were made.

B. Calculation

These notions can be made more quantitative by a model implied by that discussed above, which calculates the probability of different vintages of the measurement at a given point. The probability that the vintage of the data for a given point is zero is $q \equiv NW^2/A = 1 - p_1$, i.e., the probability that it has

just been measured. The probability that the vintage is 1 unit is the product of the probability that it was measured 1 unit before and the probability that it has not been measured since: $p_1 \cdot q$. The probability of two units is $p_1 \cdot q^2$. In general the probability of vintage s is

$$P(s) = p_1 \cdot q^s. \tag{6}$$

Figure 1 shows the probability as a function of s for N = 10 satellites over the whole earth with a 1,000 km swath. The normalization of P can be checked by

$$\Sigma_{s=0}^{\infty} P(s) = \Sigma_s\, p_1 \cdot q^s = p_1 \cdot \Sigma_s\, q^s = p_1 \cdot 1/(1-q) = 1. \tag{7}$$

The average vintage can also be computed as

$$\begin{aligned} \langle s \rangle &\equiv \Sigma_{s=0}^{\infty} P(s)\, s = \Sigma_s\, p_1 \cdot q^s\, s = p_1 \cdot \Sigma_s\, q^s\, s \\ &= p_1 \cdot q\, d/dq\, \Sigma_s\, q^s = p_1 \cdot q\, d/dq\, 1/(1-q) \\ &= p_1 \cdot q\, 1/(1-q)^2 = p_1 \cdot (1-p_1)\, 1/p_1^2 = (1-p_1)/p_1. \end{aligned} \tag{8}$$

Two limits are obvious. If $p_1 \to 1$, i.e., all cells are measured each step, $\langle s \rangle \to 0$. If $p_1 \to 0$, no cells are measured, $\langle s \rangle \to \infty$.

Each step takes time t = W/V, so $\langle s \rangle$ corresponds to a time

$$\begin{aligned} \langle T \rangle &= (1-p_1)\, 1/p_1\, W/V = (1-p_1)\, (A/NW^2)\, W/V \\ &= (1-NW^2/A)\, A/NWV, \end{aligned} \tag{9}$$

which, apart from the small $1 - p_1$ correction, is the same as the zero-overlap T_o of Eq. (1). The interpretation of this result is that a given constellation will cover the whole earth in T_o, but that the average vintage of the resulting data will be the average $\langle T \rangle$ of Eq. (9).

The average data vintage as a function of constellation parameters is shown in Fig. 2. For 5 satellites with a 50 km swath the average vintage is about 80 hours, 3 1/3 days. But by 20 satellites and a 200 km swath the vintage drops to about 5 hours. The same result would be obtained for 5 satellites with a 800 km swath. All of these results assume that the sensors work on both the light and dark sides of the Earth. For visible sensors, coverage times would be roughly doubled.

C. Inclination

Constellation inclinations impact the value of overlaps. Polar orbits mainly produce overlap over the poles, which may not

be useful. Inclined orbits produce overlaps at lower latitudes, which may be desirable, since it leads to more current information. Restricting the area of coverage, which is possible with inclined orbits, reduces the area covered proportionally, which is equivalent to increasing the satellites' swath or number.

IV. SUMMARY AND CONCLUSIONS

This note derives a phase-space estimate of the overlap in satellite coverage and evaluates its impact on the time for a constellation to cover some specified area. The satellites' motion is treated as random in the calculation of the overlaps. Enough passes are prescribed to assure that an adequate probability of observing each area is accumulated. For 0.9-0.99 probabilities of coverage, overlaps increase the time for coverage by factors of 2-4 over no-overlap estimates.

This model also gives the probability of different vintages of data. If a given constellation covers the whole Earth in the no-overlap time T_o, the average vintage of the data over the earth will then be the average $\langle T \rangle$, which is essentially the same as T_o. Overlap over the poles might be wasteful, but overlap in areas of interest by inclined orbits just causes measurements to be more current in areas of interest.

Fig. 1 Data vintage distribution

N=10 satellites, full earth coverage

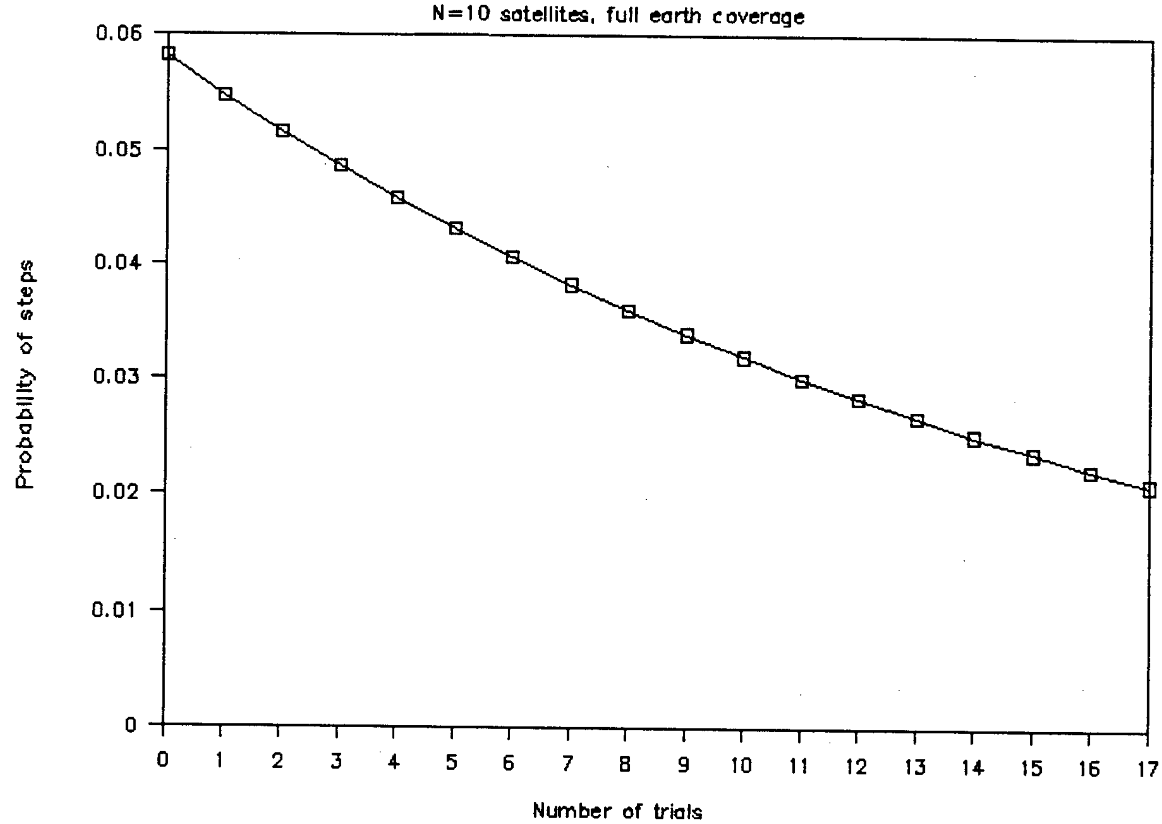

Fig. 2 Average vintage of data

full earth coverage